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STEADY MOTIONS OF A RIGID UNBALANCED ROTOR IN NON-LINEAR ELASTIC BEARINGS†

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Steady motions of a dynamically and statically unbalanced rotor, mounted in elastic bearings with cubic non-linearity, are considered, assuming the presence of resistance forces of a varying nature and that the rotor possesses four degrees of freedom. The possible types of steady motions are analysed, the conditions for the onset of self-excited vibrations are obtained and the basic characteristics of self-excited vibration modes derived. © 2002 Elsevier Science Ltd. All rights reserved.

It has been shown [1–8] that a rigid rotor rotating in elastic bearings, when the rotation frequency increases without limit, possesses a self-centring property similar to that of a flexible Laval shaft. However, these studies as a rule do not discuss the possibility of the onset of instability, self-excited vibrations, etc. In this paper the behaviour of a rigid body in elastic bearings will be investigated in detail.

1. EQUATIONS OF MOTION OF ROTORS

Consider a statically and dynamically unbalanced absolutely rigid rotor of mass M and length L, mounted vertically in two stationary elastic bearings, in such a way that its centre of mass is the same distance from each bearing. The rotor possesses dynamic symmetry, its moment of inertia about the axis of symmetry is A, and its equatorial moments of inertia are equal to B.

By static imbalance e we mean the displacement of the centre of mass from the axis of rotation of the motor, and by dynamic imblance δ we mean the angle between the axis of dynamic symmetry and the axis of rotation of the rotor. The angle between the plane passing through the axis of rotation and the centre of mass and the plane that contains the angle δ will be denoted by ε.

As regards the bearings, it is assumed that their pliability has central symmetry and the reactions of the bearings have only radial components. We shall consider the case of a rigid characteristic of the restoring force, containing a cubic term

$$\mathbf{P}_j = -\mathbf{S}_j(a_0 + a_1 \mid S_j \mid^2), \quad j = 1,2 \tag{1.1}$$

where $\mathbf{S}_j$ is the vector of displacement of the appropriate bearing from the equilibrium position, and a_0 and a_1 are positive real constants characterizing the elasticity of the bearings.

The rotor is set in motion by a motor of unlimited power, capable of maintaining a constant angular velocity of rotation ω. When investigating the steady motions of the rotor it is assumed that the rotor is subject to external friction forces $\mathbf{R}_j^e$ proportional to the absolute velocity of motion of the centre of the bearing, internal friction force $\mathbf{R}_j^i$ proportional to the relative velocity of motion of the centre of the bearing, which arise in the bearings because of the oily film partly carried by the rotor [9], as well as resistance force $\mathbf{R}_j^m$ proportional to the velocity of radial displacement of the centre of the bearing, which arise in the rigid rotor owing to deformation of the ball bearings in the rolling bearing [5]:

$$\mathbf{R}_j^e = -\tilde{\mu}_e \dot{\mathbf{S}}_j, \quad \mathbf{R}_j^i = -\tilde{\mu}_i(\dot{\mathbf{S}}_j - i\omega \mathbf{S}_j), \quad \mathbf{R}_j^m = -\tilde{\mu}_m \dot{S}_j \mathbf{S}_j / S_j, \quad j = 1,2 \tag{1.2}$$

$$(i = \sqrt{-1}).$$

It is assumed that the displacement of the rotor along the axis of rotation can be ignored, and as generalized coordinates we choose the coordinates of the axis of symmetry of the rotor, which coincide with those of the centres of the bearings in their plane of motion. This definition of the position of a rotor in space was first introduce in [10, 11]. Under these assumptions the rotor is a mechanical system with four degrees of freedom.

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Applying the law of motion of the centre of mass and the equation of moments, we obtain a system of equations of motion for a statically and dynamically unbalanced rotor in complex variables S_1 and S_2

$$\begin{aligned}
&\frac{M}{2}(\ddot{S}_1+\ddot{S}_2)+\tilde{\mu}_e(\dot{S}_1+\dot{S}_2)+\tilde{\mu}_i(\dot{S}_1+\dot{S}_2)-i\omega\tilde{\mu}_i(S_1+S_2)+\\
&+\tilde{\mu}_m\left(|\dot{S}_1|\frac{S_1}{|S_1|}+|\dot{S}_2|\frac{S_2}{|S_2|}\right)+a_0(S_1+S_2)+a_1(|S_1|^2S_1+|S_2|^2S_2)=Me\omega^2\exp(i\omega t)\\
&B(\ddot{S}_2-\ddot{S}_1)-i\omega A(\dot{S}_2-\dot{S}_1)+\tilde{\mu}_e\frac{L^2}{2}(\dot{S}_2-\dot{S}_1)+\tilde{\mu}_i\frac{L^2}{2}(\dot{S}_2-\dot{S}_1)-i\omega\tilde{\mu}_i\frac{L^2}{2}(S_2-S_1)+\\
&+\tilde{\mu}_m\frac{L^2}{2}\left(|\dot{S}_2|\frac{S_2}{|S_2|}-|\dot{S}_1|\frac{S_1}{|S_1|}\right)+\frac{L^2a_0}{2}\left(S_2-S_1+\frac{a_1}{a_0}(|S_2|^2S_2-|S_1|^2S_1)\right)=\\
&=(B-A)\omega^2L\delta\exp(i(\omega t-\varepsilon))
\end{aligned} \tag{1.3}$$

The superscript 1 corresponds to the bearing occupying the lower position when the angular velocity vector is directed upward.

To the forced vibrations of the rotor generated by its static and dynamic imbalance there correspond direct synchronous precessions of the rotor, in the form

$$S_j=\tilde{R}_j\exp(i\omega t)\exp(i\varphi_j),\quad j=1,2 \tag{1.4}$$

where $\tilde{R}_j$ and φ_j are real constants characterizing the amplitudes and phases of the bearings respectively, $\tilde{R}_j \geqslant 0$.

We will call the precessions symmetric if the bearings describe circles of the same radius in their planes of motion. Depending on the specific surface swept out in space by the rotor's axis of rotation, we distinguish three types of precession: cylindrical, conical and hyperboloid.

Cylindrical precession is characterized by equality of the amplitudes ($\tilde{R}_1=\tilde{R}_2$) and phases ($\varphi_1=\varphi_2$). In conical precession the phases either differ by π, when the apex of the cone is situated between the bearings, or coincide, when the apex is outside the bearings and the rotor axis describes a truncated cone. In precession of the hyperboloid type, the relationships between the phases and amplitudes are arbitrary.

2. A STATICALLY AND DYNAMICALLY UNBALANCED ROTOR

Consider a rotor with two imbalances ($e\neq 0$, $\delta\neq 0$). The dimensionless form of the equations of motion of the rotor is

$$\begin{aligned}
&\ddot{s}_1+\ddot{s}_2+\mu_e(\dot{s}_1+\dot{s}_2)+\mu_i(\dot{s}_1+\dot{s}_2)-i\Omega\mu_i(s_1+s_2)+\mu_m\left(|\dot{s}_1|\frac{s_1}{|s_1|}+|\dot{s}_2|\frac{s_2}{|s_2|}\right)+s_1+s_2+\\
&+c(|s_1|^2s_1+|s_2|^2s_2)=\Omega^2\exp(i\Omega\tau)\\
&\ddot{s}_2-\ddot{s}_1-i\Omega\lambda(\dot{s}_2-\dot{s}_1)+\mu_ek(1-\lambda)(\dot{s}_2-\dot{s}_1)+\mu_ik(1-\lambda)(\dot{s}_2-\dot{s}_1)-i\Omega\mu_ik(1-\lambda)(s_2-s_1)+\\
&+\mu_mk(1-\lambda)\left(|\dot{s}_2|\frac{s_2}{|s_2|}-|\dot{s}_1|\frac{s_1}{|s_1|}\right)+k(1-\lambda)(s_2-s_1+c(|s_2|^2s_2-|s_1|^2s_1))=\\
&=d(1-\lambda)\Omega^2\exp(i(\Omega\tau-\varepsilon))
\end{aligned} \tag{2.1}$$

where we have introduced the following dimensionless variables and parameters

$$\begin{aligned}
&\tau=\omega_0t,\quad s_j=\frac{S_j}{2e},\quad \lambda=\frac{A}{B},\quad \mu_e=\frac{2\tilde{\mu}_e}{M\omega_0},\quad \mu_i=\frac{2\tilde{\mu}_i}{M\omega_0},\quad \mu_m=\frac{2\tilde{\mu}_m}{M\omega_0}\\
&\Omega=\frac{\omega}{\omega_0},\quad c=4e^2\frac{a_1}{a_0},\quad k=\frac{ML^2}{4B(1-\lambda)},\quad d=\frac{L\delta}{2e};\quad \omega_0^2=\frac{2a_0}{M}
\end{aligned} \tag{2.2}$$

The dots represent differentiation with respect to dimensionless time τ.

Note that it is always true that $k(1-\lambda) > 0$. If the rotor is a dynamically prolate body, then $\lambda < 1$ and so $k > 0$; if it is dynamically oblate, then $\lambda > 1$ and $k < 0$.

Let us consider direct synchronous hyperboloid precessions (HPs) of the rotor. Symmetric HPs ($R_1 = R_2 = R$) of a statically and dynamically unbalanced rotor can only exist in a system with external friction, when the constructive parameter ε is equal to $\pi/2$. Otherwise, only asymmetric modes exist. Symmetric and asymmetric precessions of the rotor do not depend in form on the resistance forces $\mathbf{R}^i$ and $\mathbf{R}^m$.

It is convenient to change from Ω and R, corresponding to the rotation frequency and precession amplitude, to their squares $x = \Omega^2$ and $y = \mathbf{R}^2$. Then the symmetric HPs will be defined as follows:

$$\sqrt{y} = \frac{x}{2}\left(\frac{1}{(1+cy-x)^2} + \frac{d^2}{(k(1+cy)-x)^2}\right)^{1/2} \tag{2.3}$$

$$\operatorname{tg}\varphi_1 = -\operatorname{tg}\varphi_2 = d\frac{1+cy-x}{k(1+cy)-x}, \quad \varphi_1 = -\varphi_2$$

The limiting value of the amplitude R_∞ and phase $\varphi_{1,\infty}$ for sufficiently large rotation frequencies are determined from the relations

$$\sqrt{y_\infty} = R_\infty = \frac{1}{2}\sqrt{1+d^2}, \quad \operatorname{tg}\varphi_{1,\infty} = d \tag{2.4}$$

The limiting values of the minimum radius of the hyperboloid of revolution $r = R\cos\varphi_1$ and the angle between the axis of rotation of the rotor and the vertical, $\beta = 2R\sin\varphi_1/\bar{L}(\bar{L} = L/(2e)$, will be $r_\infty = 1/2$ and $\beta_\infty = \delta$, corresponding to the phenomenon of self-centring of the rotor. Rotation occurs in which the rotor's centre of mass remains stationary and the axis of dynamic symmetry occupies the equilibrium position – the most favourable case.

If the rotor is a dynamically prolate body ($\lambda < 1$ and $k > 0$), the resonance set of the system consists of the straight lines

$$1 + cy - x = 0 \tag{2.5}$$

$$k(1 + cy) - x = 0 \tag{2.6}$$

which are skeletal for the modified amplitude-frequency characteristic (AFC) of a symmetric HP in the (x, y) plane; these lines are represented by the inclined dashed lines in Fig. 1 (the horizontal dashed line corresponds to the limiting value y_∞). In this situation, precessions of cylindrical type resonate near the set (2.5) and those of conical type, near the set (2.6).

Given a fixed value of the rotation frequency, one, three or five different symmetric HP modes may exist. By investigating the stability of the modes in the linear approximation and verifying the necessary condition for stability, it can be shown that, for a system without resistance, there are certain values of k ($k < 1/3$ and $k > 3$) such that the arcs AB and CD (Fig. 1) represent unstable symmetric HPs.

If the rotor is dynamically oblate ($\lambda > 1$ and $k < 0$), the resonance set is a single straight line, near which a cylindrical precession will resonate. Under these conditions an arc of the AFC with the mean value of the amplitude will correspond to unstable motions for any values of the parameters.

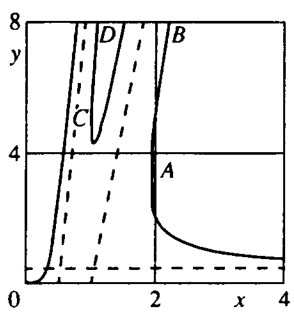

Fig. 1

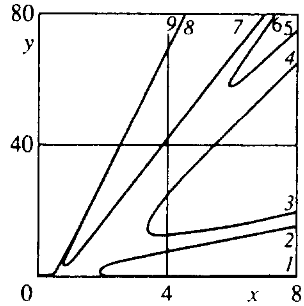

Fig. 2

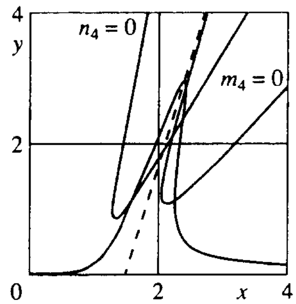

Fig. 3

We shall also consider asymmetric HPs, which develop when the axis of dynamic symmetry lies in the same plane as the axis of rotation ($\varepsilon = 0$). Note that when there are no external friction forces HPs degenerate into conical precessions, and the rotor axis may describe either a truncated cone or a cone with its apex between the bearings. It has been established by analytical means that asymmetric modes, like symmetric modes, possess the self-centring property (branch 1 in Fig. 2). Further analysis of asymmetric precessions will be based on computer simulation.

Depending on the frequency of rotation and the external resistance force, an odd number of asymmetric modes – from one to nine may exists. As the external resistance force is increased, the possible number of solutions decreases to one. Figure 2 illustrates the dependence of the squared amplitude of the first bearing on the squared rotation frequency when up to nine modes may exist. If the only forces acting on the system are external friction forces $\mathbf{R}^e$, the even branches are always unstable, while the odd branches may be either stable or unstable. At rotation frequencies for which there is instability of odd branches, stable self-excited vibrations may develop.

The internal friction forces $\mathbf{R}^i$ and resistance forces $\mathbf{R}^m$ do not have a stabilizing effect on unstable modes; moreover, the region of stable modes contracts strongly. Beginning at a certain rotation frequency, all modes become unstable except the one with the maximum amplitude of the first bearing.

3. A DYNAMICALLY UNBALANCED ROTOR

Now suppose the rotor has only dynamic imbalance ($e = 0, \delta \neq 0$), The dimensionless equations of motion for such a rotor differ formally from Eqs (2.1) in that the right-hand sides of the equations are now respectively zero and $^1/_2(1 - \lambda)\Omega^2 \exp(i\Omega\tau)$. The dimensionless variables and parameters are defined as in Section 2, except for the variable s_j and the parameter c; now $s_j = S_j/(2L\delta)$, $c = 4L^2\delta^2 a_1/a_0$.

Among the steady motions of a dynamically unbalanced rotor corresponding to direct synchronous precessions, symmetric conical precessions may exist, when the centre of mass of the rotor is stationary, and also asymmetric HPs, associated with the motion of the centre of mass, which degenerate into conical precessions when there are no viscous friction forces. If the system is not subject to viscous friction forces ($\mu_e = 0$), then, beside the aforementioned motions, symmetric HPs will also exist, as well as motions in which the amplitude of one of the bearings is zero. Under these conditions, just as in the case of a totally unbalanced rotor, the modes are independent in form of the internal friction forces $\mathbf{R}^i$ and resistance forces $\mathbf{R}^m$.

We shall now consider the motions of most interest in greater detail.

Symmetric conical precessions (SCPs), when the centre of mass of the rotor is stationary ($s_1 = -s_2 = -s$), are defined by the relations

$$\sqrt{y}\sqrt{(k(1+cy)-x)^2+\mu_e^2k^2x}=\frac{1}{4}x, \quad \operatorname{tg}\varphi=-\frac{\mu_e k\sqrt{x}}{k(1+cy)-x} \tag{3.1}$$

If the rotor is a dynamically prolate body ($\lambda < 1, k > 0$), the modified AFC in the (x, y) plane, shown in Fig. 3, has a skeletal straight line (2.6), represented by the dashed line in Fig. 3. The limiting value of the amplitude at large values of the rotation frequency is $R_\infty = 1/4$.

The limiting value (at large values of the rotation frequency) of half the aperture angle of the cone swept out in space by the rotor's axis of rotation will be equal to δ, implying self-centring of the rotor. The rotor will rotate in such a way that its axis of dynamic symmetry tends to occupy an equilibrium position.

Depending on the parameters of the system, one or three different SCP modes may exist for different angular velocities. When investigating stability in the first approximation, the characteristic equation of a linear system may be expressed in the form $MN = 0$, where M and N are polynomials of degree four in the characteristic number p

$$M=m_0p^4+m_1p^3+m_2p^2+m_3p+m_4, \quad N=n_0p^4+n_1p^3+n_2p^2+n_3p+n_4 \tag{3.2}$$

Using the Hurwitz criterion, we conclude that the conditions for stability for a dynamically prolate rotor reduce to the inequalities

$$m_3>0,\ m_4>0,\ n_3>0,\ n_4>0,\ \Delta_3^{(M)}>0,\ \Delta_3^{(N)}>0 \tag{3.3}$$

where $\Delta_3^{(M)}$ $\Delta_3^{(N)}$ are the principal diagonal minors of order three of the Hurwitz determinants for the polynomials M and N.

The coefficients m_4 and n_4 are the products of the roots of the respective polynomials, and the sets $m_4 = 0$ and $n_4 = 0$ are bifurcational for the system and determine the stability boundary of SCPs. In the (x, y) plane, these sets are hyperbolae (Fig. 3). Bifurcations at the boundary $m_4 = 0$ correspond to the appearance or disappearance of SCPs, when the centre of mass of the rotor remains stationary; bifurcations at the boundary $n_4 = 0$ correspond to the appearance or disappearance of asymmetric precessions due to motion of the centre of mass. Instability of this kind may be classed with the phenomenon of spatial instability of forced vibrations, which has been investigated for quasi-linear systems [2].

If one considers a rotor with two imbalances, without taking resistance forces into consideration, the AFC of a symmetric HP will have two resonance zones, corresponding to resonances of cylindrical and conical precessions. When the rotor is considered as a system with four degrees of freedom, but possessing only dynamic imbalance, a region of spatial instability will also develop near the resonance set (2.5) for cylindrical precessions of a rotor with two imbalances. A redistribution of vibrations occurs, and the arc of the AFC where SCPs are unstable ($n_4 < 0$) will correspond to HPs or, in some cases, to other stable modes.

The sets $m_4 = 0$ and $n_4 = 0$ do not depend on the internal friction forces $\mathbf{R}^i$ and resistance forces $\mathbf{R}^m$, and if the only forces acting on the rotor are those of external resistance, all modes except those within the hyperbolae $m_4 = 0$, $n_4 = 0$ are stable.

If the rotor is dynamically oblate, the modified AFC increases monotonically and approaches the limiting value $y_\infty = R_\infty^2 = 1/16$. The phenomenon of spatial instability may also be observed in a dynamically oblate rotor.

Asymmetric HPS, unlike SCPs, do not always exist. The system of differential equations of a dynamically unbalanced rotor is such that, if a mode with amplitudes (R_1, R_2) exists, then a mode with amplitudes (R_2, R_1) will also exist. Depending on the parameters of the system and the rotation frequency, the theoretically possible number of pairs of solutions may vary from one to four. Under these conditions, if there are only external resistance forces, it may be shown by numerical means that asymmetric HPs may be either stable or unstable in the spatial instability region of SCPs. For rotation frequencies such that asymmetric HPS are unstable, one obtains in the $(R, \dot{R})$ plane of eight-dimensional phase space "looping" curves in a bounded region, similar to strange attractors.

Due to the internal friction forces $\mathbf{R}^i$, as well as the resistance forces $\mathbf{R}^m$, SCPs of the rotor which are stable when there are only external friction forces $\mathbf{R}^e$ will become unstable from some angular velocity of rotation on, which excludes the self-centring effect. It has been established numerically that the modes arising in that case are associated with motion of the centre of mass and are self-excited vibrations.

When the rotor is subject to external and internal resistance forces, the modes thus obtained may be sought approximately in the form

$$s_j = R_j \exp(i\Omega\tau)\exp(i\varphi_j) + r_j \exp(i\Omega_1\tau) \tag{3.4}$$

where R_j and φ_j are the amplitudes and phases, respectively, of the forced vibrations at velocity Ω, and r_j and Ω_1 the amplitude and frequency, respectively, of self-excited vibrations [13].

If the centre of mass of the rotor describes a circle, the solution may be sought as the superposition of two motions: an SCP ($R_1 = R_2 = R$ and $\exp(i\varphi_2) = -\exp(i\varphi_1) = \exp(i\varphi)$) and symmetric self-excited vibrations ($r_1 = r_2 = r$).

Applying the harmonic balance method, we obtain approximate equations for the quantities R, r, Ω_1, φ

$$r(1 + c(r^2 + R^2) - \Omega_1^2) = 0, \quad r(\Omega_1(\mu_e + \mu_i) - \Omega\mu_i) = 0$$

$$R(1-\lambda)((k(1 + c(R^2 + 2r^2)) - \Omega^2)\sin\varphi + \mu_e k\Omega\cos\varphi) = 0 \tag{3.5}$$

$$(1-\lambda)(R(k(1 + c(R^2 + 2r^2)) - \Omega^2)\cos\varphi - \mu_e k\Omega R\sin\varphi - \tfrac{1}{4}\Omega^2) = 0$$

Equations (3.5) yield the amplitudes of self-excited vibrations and forced vibrations as functions of the rotation frequency and determine the boundary of soft excitation of self-excited vibrations

$$\Omega_e = (1 + \mu_e/\mu_i)\sqrt{1 + 2cR^2} \tag{3.6}$$

Note that as the rotation frequency is increased, the amplitude of self-excited vibrations also increases, possible leading in the final analysis to failure of the bearings and breakdown of the rotor.

If the rotor is a system with two degrees of freedom, self-excited vibrations must be sought in the form (3.4), when $s_1 = -s_2 = -s$. Such modes exist only for a dynamically prolate rotor ($\lambda < 1$ and $k > 0$). Although, formally speaking, this form of the solution satisfies the system of differential equations of a dynamically unbalanced rotor with four degrees of freedom, numerical integration leads to modes associated with motion of the centre of mass.

4. A STATICALLY UNBALANCE ROTOR

Among the steady motions of a statically unbalanced rotor ($e \neq 0$, $\delta = 0$) corresponding to direct synchronous precessions, cylindrical precessions may exist and, in the case of a dynamically prolate rotor ($\lambda < 1$, $k > 0$), asymmetric hyperboloid precessions. If there are no external resistance forces, a dynamically prolate rotor may develop symmetric hyperboloid precessions, as well as asymmetric precessions of conical type, when one of the bearings is stationary. The possible types of precession may be investigated along the same lines as our investigation of the analogous modes of a dynamically unbalanced rotor. We merely note that the self-centring property is characteristic for cylindrical precessions; for a dynamically prolate rotor, the phenomenon of spatial instability may arise, and the corresponding modes are associated with swinging of the rotor's axis of rotation. Internal friction forces $\mathbf{R}^i$ and resistance forces $\mathbf{R}^m$ exclude the phenomenon of self-centring in the rotor, and the self-excited vibrations that appear constitute a plane-parallel motion.

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